Topology Optimization for Rotating Machinery

Hiroki Sakamoto,
Takashi Abe

Keywords Finite element method, Topology optimization, Electromagnetic field analysis, IPM motor

Abstract

The market recently calls for higher performance of electromagnetic equipment. To meet such market demands, we are advancing design technologies by focusing on shape design. We newly developed a shape optimization technology that is attracting interest. This technology refers to a method where shape topology is treated as a design variable seeking optimization. It has the highest design freedom among other shape optimization methods. As a result, it produces unique out-of-the-box design ideas. We applied this technology into the shape design of a rotor core of rotating machinery. As a result, we obtained a shape design with improved characteristics and verified the validity of this development approach.

1 Preface

Against the background of environmental protection and energy-saving challenges, the market calls for higher performance rotating machinery. Such market demand is strong against the interior Permanent Magnet Synchronous Motor (IPMSM). The IPMSM is widely used as a driving motor for electric vehicles which requires high torque and high efficiency. The rotor core shape, which is an essential element to determine IPMSM performance, consists of electromagnetic steel sheets, permanent magnets, and air regions. The degree of design freedom is high. In the rotor core shape design, the design quality greatly depends on the designer's design knowledge, experience, and past performance. Optimal designing is not easy to attain. In order to realize optimal design for the rotor core shape, we promoted a joint research program with a graduate school course at Hokkaido University. The program specializes in optimization technology of the topology. Topology optimization is attracting general interest as an optimization approach. This paper introduces the topology optimization technology, example of application, and the result of actual measurements based on the result of our joint research.

2 Topology Optimization Technology(1)

Topology optimization is conducted in a specified fixed design domain to solve material distribution issues suitable for designs that require vacant placement and number determination.

Since the shape is changed using the topology as a design variable, this approach is suitable for rotor core design of the IPMSM. Such design is required to determine the air regions layout and the number of regions.

In the case of topology optimization, a challenge of designing is reviewed as the object domain Ω distributed in the design domain D in order to obtain an optimal distribution of the object domain Ω and the cavity domain $D\Omega$ within the design domain D. The shape and mode of the object domain Ω within the design domain D are expressed by using the characteristic function $\chi\Omega(x)$ below.

$$\chi\Omega(x)=\begin{cases} 1 \text{ for } x\in\Omega \\ 0 \text{ for } x\in\Omega/D \end{cases} \quad\cdots\cdots\cdots\cdots (1)$$

Where, x denotes the coordinates of any point within D. If $\chi\Omega(x)$ is 1, a material is present at that point. If it is 0, the material is not present there. The value $\chi\Omega(x)$ is defined in the continuous system, but it must be replaced by the discrete system when evaluating with finite elements.

In electromagnetic field analysis, replacement of characteristic function generally uses an approach called the ON-OFF method(2) and each element in the design domain is optimized by the probabilistic optimization method.

Since this approach makes it possible to change the element status freely, it may create a shape which is difficult to realize in view of technical engineering or it might increase the computation time for optimization work due to the vast design variables counting on element number in the design domain. This is the challenge of the ON-OFF method.

Regarding our optimization approach, the characteristic function $\chi\Omega(x)$ is approximated by using normalized Gaussian function network (NGnet)(3)(4). The NGnet is a kind of approximator that generates an output by accumulation of normalized Gaussian functions that smoothly change in space. Accordingly, it becomes easier to obtain a technically practical solution and a design variable becomes a parameter of NGnet. This approach provides a feature that design variables are decreased.

$$y(x)=\sum_{i=1}^{N} w_i b_i(x) \quad \cdots\cdots (2)$$

$$b_i(x)=\frac{G_i(x)}{\sum_{k=1}^{N} G_k(x)} \quad \cdots\cdots (3)$$

$$G_k(x)=\frac{1}{(2\pi)^{D/2}\,|\Sigma_k|^{1/2}}\exp\left\{-\frac{1}{2}(x-u_k)^{T}\,\Sigma_k^{-1}\,(x-u_k)\right\} \quad \cdots\cdots (4)$$

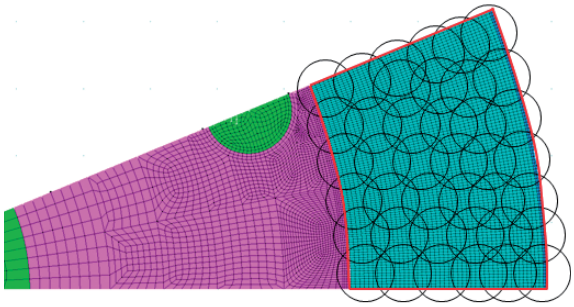
Where, N is a numerical value of the Gaussian function, D is the dimension of Input x, u_k and Σ_k are a center vector and a covariance matrix of Gaussian function k, and w_i is a coupling matrix mass of normalized Gaussian function $b_i(x)$.

Using Output $y(x)$ of the NGnet, the characteristic function dispersed by Element e is defined by the Expression below.

$$\chi\Omega(x_e)=\begin{cases} 1 & y(x_e)\geq 0 \\ 0 & y(x_e)<0 \end{cases} \quad \cdots\cdots (5)$$

Where, Value x_e is the coordinate value of Element e at the center of gravity.

Fig. 1 shows the layout of the Gaussian base within the design domain. For topology optimization, the Gaussian base is manually and densely arranged within the design domain to change the Coupling weight w_i. By this treatment, each NGnet can be changed to let the overall shape vary. In this way, the value of Coupling weight w_i is optimized by the probabilistic optimization method so that topology optimization can be conducted. This time, we explained two types of materials, air and electromagnetic steel sheets. If multiple NGnet items are taken into consideration, the effect of this approach can be extended to the optimization of multiple materials, such as electromagnetic steel sheets, magnets, and air regions, by superpositioning.



Red frame: Design domain
Black circle: Gaussian base

Fig. 1 **Layout of Gaussian Base within Design Domain**

The layout of the Gaussian base is shown. 22.5° angle of the rotor core is assumed to be within the design domain.

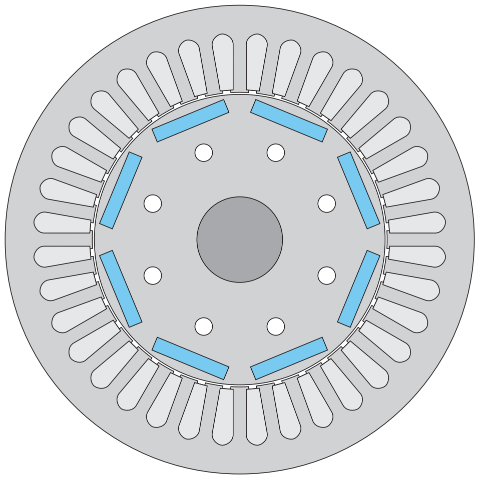


Fig. 2 **Shape of Original Model's Cross-Section**

The shape of a cross-section of an 8-pole 36-slot IPMSM is shown. This is the target of optimization.

3 Example of Optimization

Optimization was carried out on an 8-pole 36-slot IPMSM ("original model" hereafter). **Fig. 2** shows a shape of original model's cross-section and **Table 1** shows the motor specifications. For this time, optimization was conducted against electromagnetic steel sheets, magnets, and air gaps so that the mean torque can be maximized while the

Table 1 **Motor Specifications**

Motor specifications are shown.

Item	Specifications
Stack length	50 mm
Output power	1.5 kW
Rotate speed	1500 min^{-1}
No. of turns	31 turn

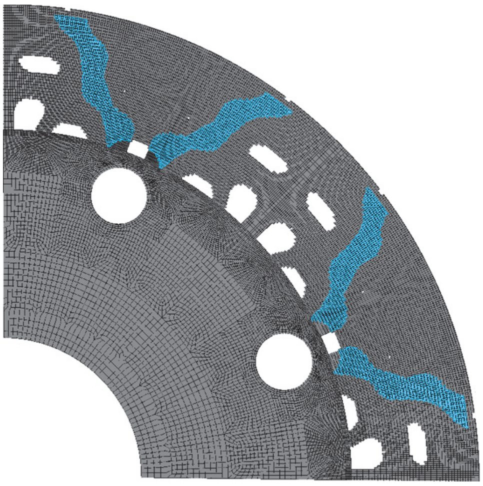


Fig. 3 **Optimized Shape for Magnet Volume Reduction**

We obtained a shape design where the magnet volume is reduced by 46% yet the average torque is increased by 4% (compared with the original model). This shape design, however, is not optimal for manufacturing.

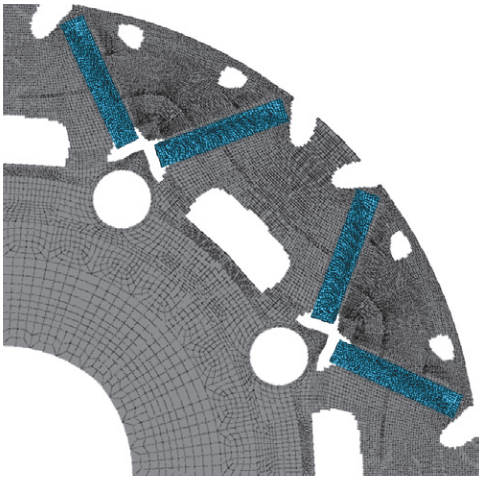


Fig. 4 **Optimized Model Shape**

Magnets in Fig. 3 were rearranged into a straight shape. A resultant shape is shown.

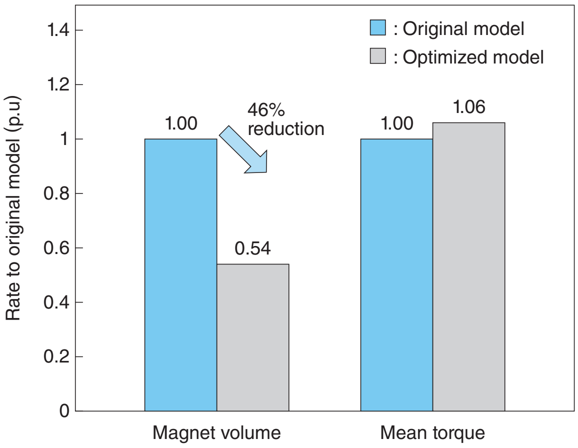


Fig. 5 **Result of Optimization**

We obtained a shape design where the magnet volume is reduced by 46% yet the average torque is increased by 6%.

Fig. 6 **Shape of Prototyped Rotor Core**

We simplified the rotor core design in the areas which will not influence the magnetic characteristics.

magnet volume is being reduced.

Fig. 3 shows an optimized shape for the magnet volume reduction. Its maximum mean torque was more than that of the original model. The magnet volume was reduced by 46% compared with that of the original model. For actual production, however, this design is difficult to realize the rippling shape of magnets. As a result, we changed the shape of magnets to a rectangle manually for easier manufacturing. In such a case, however, we had some concern about the degradation of magnet characteristics. Thus, we conducted optimization work again, with magnets fixed. Fig. 4 shows the optimized model shape and Fig. 5 shows the result of optimization.

At the final stage of development, a mockup was produced to confirm the effectiveness of the optimized model and torque characteristics were measured. Fig. 6 shows the shape of prototyped rotor core. Regarding the mockup shape of the optimized model, however, some parts not affecting the magnetic characteristics were changed in its design, considering the ease of manufacturing.

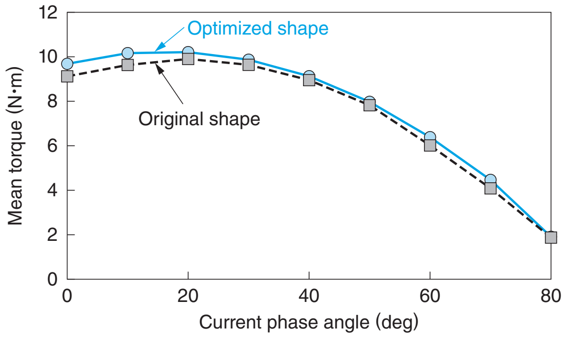


Fig. 7 Relationship between Mean Torque and Current Phase Angle (Actually Measured)

Even actual measurements indicate that the mean torque is improved.

Fig. 7 shows the relationship between mean torque and current phase angle (actually measured). Despite almost half of the volume of magnets were reduced from the original model, the obtained mean torque exceeded that of the original. For this optimization program, almost no manpower was used, except when we studied the means of easing manufacturing. We confirm that an effective shape can be obtained through the topology optimization.

4 Design Guide Method

Due to manufacturing requirements as described in **Section 3**, the result of optimization cannot be utilized immediately in some cases. In this case, it is necessary to modify the resultant shape so that characteristics are not degraded by the designer. For this purpose, we developed an approach ("Design Guide Method" hereafter) to help the designer to accomplish shape modification through the visualization of sensitivity against the characteristics of the shape. In this section, we explained the method on the visualization of the shape after the change of characteristics. This is made according to the sensitivity.

For the Design Guide Method, Characteristic $T(w)$ is defined as a function of Coupling Weight w of the NGnet in order to determine the changing direction of the shape when $T(w)$ is increased or decreased. Change of T dT caused by a change in w in Space w is defined by Expression (6) below.

$$dT=\sum_{i=1}^{n}\frac{\partial T}{\partial w_i}dw_i=\nabla_w T\cdot dw \cdots\cdots\cdots\cdots (6)$$

Where, $\nabla_w\phi$ is given by Expression (7) below.

$$\nabla_w\phi=\nabla_w\sum_{i=1}^{n}w_i b_i=[b_1\ b_2\cdots b_n]^t \cdots\cdots\cdots\cdots (7)$$

A change of ϕ $\delta\phi$ when it changes in the direction of $\nabla_w T$ is given by Expression (8) below.

$$\begin{aligned}\delta\phi(x)&=\phi(w+\nabla_w T)-\phi(w)\\&=\phi(w)+\varepsilon\nabla_w T\cdot\nabla_w\phi-\phi(w)\\&=\varepsilon\nabla_w T\cdot\nabla_w\phi\\&=\varepsilon\sum_{i=1}^{n}\frac{\partial T}{\partial w_i}b_i(x) \cdots\cdots\cdots\cdots (8)\end{aligned}$$

ε : fine constant

In this case, Characteristic T is changed in the direction of the target by increasing the physical property when $\delta\phi(x)>0$ on the boundary between magnetic body and air, or by decreasing the physical property when $\delta\phi(x)<0$.

Now the method of determining the permissible value of characteristics is considered. When a change occurs in the direction of $\nabla_w T$, Change in Characteristic δT in the changing direction of characteristics in Space w is given by the following Expression:

$$\begin{aligned}\delta T(w)&=T(w+\varepsilon\nabla_w T)-T(w)\\&=T(w)+\varepsilon\nabla_w T\cdot\nabla_w T-T(w)\\&=\varepsilon\nabla_w T\cdot\nabla_w T\\&=\varepsilon\sum_{i=1}^{n}\frac{\partial T}{\partial w_i}\frac{\partial T}{\partial w_i} \cdots\cdots\cdots\cdots (9)\end{aligned}$$

From Expression (9), it is known that the amount of change in characteristics can be controlled by ε. A suitable value ε can be determined by gradually increasing ε, starting from a sufficiently small ε. For example, when considering a change in the direction of sensitivity where 5% characteristics are assumed tolerable, calculation is continued by changing ε close to the limits where Expression (10) is satisfied.

$$\frac{\delta T(w)}{T(w)}<0.05 \cdots\cdots\cdots\cdots (10)$$

If shape depiction is performed with the use of the output function shown by Expression (11), the shape after characteristic change can be made visible.

$$\phi(w+\varepsilon\nabla_w T) \cdots\cdots\cdots\cdots (11)$$

Fig. 8 shows an example of visualization by the Design Guide Method. Changes can be evaluated if visualization is conducted with a black line

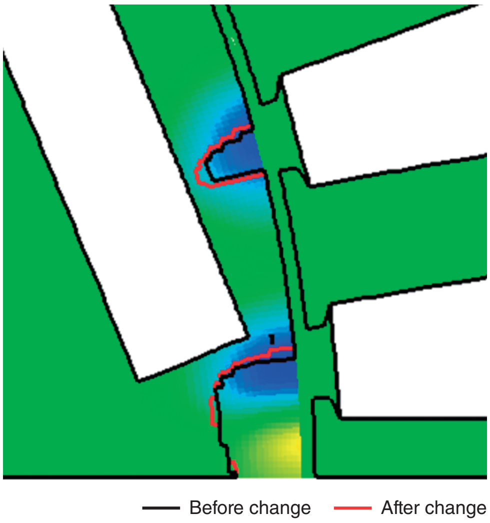


Fig. 8 Example of Visualization by the Design Guide Method

Condition before change (optimized shape) is indicated by a black line. The condition after change (characteristic permissible shape) is indicated by a red line. Shape adjustment in this area will have a minimal impact on torque.

indicating the phase before change (optimized shape) and a red line depicting the phase after change (characteristic permissible shape). When this approach is adopted, a wanted shape modification mode can be chosen according to the purpose of application such as improvement of characteristics, minimization of influence, and improvement of manufacturability.

5 Postscript

This paper introduced our program on topology optimization technologies. Such technologies are recently attracting general interest. Going forward, we will continue to develop cutting-edge analytical technologies. By using such technologies, we will design and develop new products that contribute to society. Last but not least, we would like to express our gratitude to Professor Hajime Igarashi, Course of Computer Science and Information Technology, the Graduate School, Hokkaido University for his valuable suggestions, guidance, and theoretical reviews during our technical development of topology optimization.

- All product and company names mentioned in this paper are the trademarks and/or service marks of their respective owners.

《References》

(1) Shinji Nishiwaki, Kazuhiro Izui, Noboru Kikuchi: [Topological Optimization], Maruzen Publishing, 2013 (in Japanese)

(2) Watanabe, F. Campelo, Y. Iijima, K. Kawano, T. Matsuo, T. Mifune, T., and H. Igarashi, "Optimization of inductors using Evolutionary algorithms and its experimental validation," IEEE Trans. on Magn., Vol.46, No.8, pp.3393-3396, 2010.

(3) Takahiro Sato, Hajime Igarashi, Shinya Takahashi, Shou Uchiyama, Keisuke Matsuo, Daiki Matsuhashi: "Optimization of Rotor Shape for Implanted Permanent Magnet Synchronous Motors by Topological Optimization," Journal of the Institute of Electrical Engineers, D, Journal of Industrial Applications, Vol.135, No.3, pp.291-298, 2015 (in Japanese)

(4) Shintaro Furui, Hajime Igarashi, Hiroki Sakamoto, Takashi Abe, Kazuya Ogura: "Topological Optimization of IPM Motors in Consideration of Losses," Static Apparatus and Rotary Machine Joint Study Group of the Institute of Electrical Engineers, SA-16-023, PM-16-023, 2016 (in Japanese)